

**CH5 – LAWS OF MOTION**

1. Give the magnitude and direction of the net force acting on

(a) a drop of rain falling down with a constant speed

(b) a cork of mass 10 g floating on water

(c) a kite skillfully held stationary in the sky

(d) a car moving with a constant velocity of 30 km/h on a rough road

(e) a high-speed electron in space far from all material objects, and free of electric and magnetic fields.

**Solution**

(a) The raindrop is falling at a constant speed. Therefore, acceleration will become zero. When acceleration is zero, the force acting on the drop will become zero since  $F = ma$ .

(b) The cork is floating on water, which means the weight of the cork is balanced by the upthrust. Therefore, the net force on the cork will be zero

(c) Since the car moves with a constant velocity, the acceleration becomes zero. Therefore, the force will be zero.

(d) The net force acting on the high-speed electron will be zero since the electron is far from the material objects and free of electric and magnetic fields.

2. A pebble of mass 0.05 kg is thrown vertically upwards. Give the direction and magnitude of the net force on the pebble,

(a) during its upward motion

(b) during its downward motion

(c) at the highest point where it is momentarily at rest. Do your Solutions change if the pebble was thrown at an angle of  $45^\circ$  with the horizontal direction? Ignore air resistance.

**Solution**

(a) During the upward motion of the pebble, the acceleration due to gravity acts downwards, so the magnitude of the force on the pebble is

$$F = mg = 0.05 \text{ kg} \times 10 \text{ ms}^{-2} = 0.5 \text{ N}$$

The direction of the force is downwards

(b) During the downward motion also the magnitude of the force will be equal to 0.5 N and the force acts downwards

(c) If the pebble is thrown at an angle of  $45^\circ$  with the horizontal direction, it will have both horizontal and vertical components of the velocity. At the

highest point, the vertical component of velocity will be zero but the horizontal component of velocity will remain throughout the motion of the pebble. This component will not have any effect on the force acting on the pebble. The direction of the force acting on the pebble will be downwards and the magnitude will be 0.5 N because no other force other than acceleration acts on the pebble.

**3. Give the magnitude and direction of the net force acting on a stone of mass 0.1 kg,**

**(a) just after it is dropped from the window of a stationary train**

**(b) just after it is dropped from the window of a train running at a constant velocity of 36 km/h**

**(c) just after it is dropped from the window of a train accelerating with  $1 \text{ m s}^{-2}$**

**(d) lying on the floor of a train which is accelerating with  $1 \text{ m s}^{-2}$ , the stone being at rest relative to the train. Neglect air resistance throughout.**

**Solution**

**(a) Mass of stone = 0.1 kg**

**Acceleration =  $10 \text{ ms}^{-2}$**

**Net force,  $F = mg = 0.1 \times 10 = 1.0 \text{ N}$**

**The force acts vertically downwards**

**(b) The train moves at a constant velocity. Therefore, the acceleration will be equal to zero. So there is no force acting on the stone due to the motion of the train. Therefore, the force acting on the stone will remain the same (1.0 N)**

**(c) When the train accelerates at  $1 \text{ m/s}^2$ , the stone experiences an additional force of  $F' = ma = 0.1 \times 1 = 0.1 \text{ N}$ . The force acts in the horizontal direction. But as the stone is dropped, the force  $F'$  no longer acts and the net force acting on the stone  $F = mg = 0.1 \times 10 = 1.0 \text{ N}$ . (vertically downwards).**

**(d) As the stone is lying on the train floor, its acceleration will be the same as that of the train.**

**Therefore, the magnitude of the force acting on the stone,  $F = ma = 0.1 \times 1 = 0.1 \text{ N}$ .**

**It acts along the direction of motion of the train.**

**4. One end of a string of length  $l$  is connected to a particle of mass  $m$  and the other to a small peg on a smooth horizontal table. If the particle moves in a circle with speed  $v$  the net force on the particle (directed towards the centre) is :**

(i) T

(ii)  $T - mv^2/l$ (iii)  $T + mv^2/l$ 

(iv) 0

T is the tension in the string. [Choose the correct alternative].

**Solution (i) T**

The net force acting on the particle is T, and it is directed towards the centre. It provides the centripetal force required by the particle to move along a circle.

**5. A constant retarding force of 50 N is applied to a body of mass 20 kg moving initially with a speed of  $15 \text{ m s}^{-1}$ . How long does the body take to stop?**

**Solution**

Here,

Force = - 50 N (since it is a retarding force)

Mass  $m = 20 \text{ kg}$

$v = 0$

$u = 15 \text{ m s}^{-1}$

Force  $F = ma$

$a = F/m = -50/20 = -2.5 \text{ ms}^{-2}$

Using the equation,  $v = u + at$

$0 = 15 + (-2.5) t$

$t = 6 \text{ s}$

**6. A constant force acting on a body of mass 3.0 kg changes its speed from  $2.0 \text{ ms}^{-1}$  to  $3.5 \text{ ms}^{-1}$  in 25 s. The direction of the motion of the body remains unchanged. What is the magnitude and direction of the force?**

**Solution:**

Given,

Mass,  $m = 3.0 \text{ Kg}$

$u = 2.0 \text{ m/s}$

$v = 3.5 \text{ m/s}$

$t = 25 \text{ s}$

$F = ma$

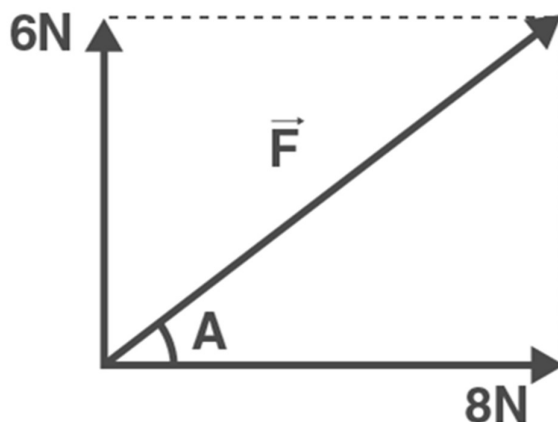
$F = m [(v-u)/t]$  (since  $a = (v-u)/t$ )

$F = 3 [(3.5 - 2)/25] = 0.18 \text{ N}$

The force acts in the direction of motion of the body

7. A body of mass 5 kg is acted upon by two perpendicular forces 8 N and 6 N. Give the magnitude and direction of the acceleration of the body.

Solution:



Given,

Mass,  $m = 5 \text{ kg}$

Force,  $F_1 = 8\text{N}$  and  $F_2 = 6\text{N}$

The resultant force of the body

Acceleration,  $a = F/m$

$a = 10/5 = 2\text{m/s}$  in the direction of the resultant force

The direction of the acceleration,

$\tan \beta = 6/8 = 0.75$

$\beta = \tan^{-1} (0.75)$

$\beta = 37^\circ$  with 8N

8. The driver of a three-wheeler moving at a speed of 36 km/h sees a child standing in the middle of the road and brings his vehicle to rest in 4.0 s just in time to save the child. What is the average retarding force on the vehicle? The mass of the three-wheeler is 400 kg, and the mass of the driver is 65 kg.

Solution:

Given,

Initial velocity,  $u = 36 \text{ km/h}$

Final velocity,  $v = 0$

Mass of the three-wheeler,  $m_1 = 400 \text{ Kg}$

Mass of the driver,  $m_2 = 65 \text{ Kg}$

Time taken to bring the vehicle to rest = 4.0 s

Acceleration,  $a = v - u/t = (0 - 10)/4 = -2.5 \text{ m/s}$

$$\text{Now, } F = (m_1 + m_2) / a = (400 + 65) \times (-2.5) \\ = - 1162.5 \text{ N} = - 1.16 \times 10^3 \text{ N}$$

The negative sign shows that the force is retarding

**9. A rocket with a lift-off mass of 20,000 kg is blasted upwards with an initial acceleration of  $5.0 \text{ ms}^{-2}$ . Calculate the initial thrust (force) of the blast.**

**Solution:**

Given,

Mass of the rocket,  $m = 20000 \text{ kg} = 2 \times 10^4 \text{ kg}$

Initial acceleration =  $5 \text{ ms}^{-2}$

$$g = 9.8 \text{ m/s}^2$$

The initial thrust (force) should give an upward acceleration of  $5 \text{ ms}^{-2}$  and should overcome the force of gravity.

Thus, the thrust should produce a net acceleration of  $9.8 + 5.0 = 14.8 \text{ ms}^{-2}$ .

Using Newton's second law of motion, the initial thrust acting on the rocket

Thrust = force = mass  $\times$  acceleration

$$F = 20000 \times 14.8 = 2.96 \times 10^5 \text{ N}$$

**10. A body of mass 0.40 kg moving initially with a constant speed of  $10 \text{ ms}^{-1}$  to the north is subject to a constant force of 8.0 N directed towards the south for 30 s. Take the instant the force is applied to be  $t = 0$ , the position of the body at that time to be  $x = 0$ , and predict its position at  $t = -5 \text{ s}$ ,  $25 \text{ s}$ ,  $100 \text{ s}$ .**

**Solution:**

Given,

Mass of the body = 0.40 kg

Initial velocity,  $u = 10 \text{ m/s}$

Force,  $f = -8 \text{ N}$  (retarding force)

Using the equation  $S = ut + (\frac{1}{2}) at^2$

(a) Position at the time  $t = -5 \text{ s}$

The force starts acting on the body from  $t = 0 \text{ s}$

So the acceleration of the body when the time is  $-5 \text{ s}$  is 0

$$S_1 = (10)(-5) + (\frac{1}{2})(0)(-5)^2 = -50 \text{ m}$$

(b) Position at the time  $t = 25 \text{ s}$

The acceleration of the body due to the force acting in the opposite direction

$$\text{Acceleration, } a = F/m = -8 / 0.4 = -20 \text{ ms}^{-2}$$

$$S_2 = (10)(25) + (\frac{1}{2})(-20)(25)^2 = -6000 \text{ m}$$

(c) Position at the time  $t = 100$  s

For the first 30 sec, the body will move under the retardation of the force, and after that, the speed will remain constant.

Therefore, distance covered in 30 sec

$$S_3 = (10)(30) + \left(\frac{1}{2}\right)(-20)(30^2)$$

$$= 300 - 9000 = -8700\text{m}$$

The speed after 30 sec is

$$v = u + at$$

$$v = 10 - (20 \times 30) = 590 \text{ m/s}$$

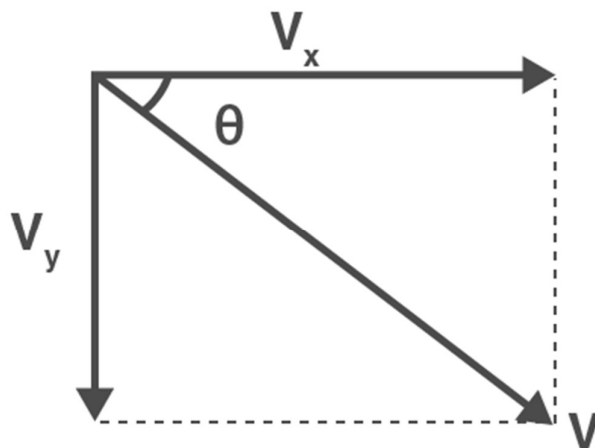
The distance covered in the next 70 sec is

$$S_4 = -590 \times 70 + \left(\frac{1}{2}\right)(0)(70)^2 = -41300 \text{ m}$$

Therefore the position after 100 sec  $= S_3 + S_4 = -8700 - 41300 = -50000\text{m}$

11. A truck starts from rest and accelerates uniformly at  $2.0 \text{ ms}^{-2}$ . At  $t = 10$  s, a stone is dropped by a person standing on the top of the truck (6 m high from the ground). What are the (a) velocity, and (b) acceleration of the stone at  $t = 11\text{s}$ ? (Neglect air resistance.)

Solution:



Initial velocity,  $u = 0$

Acceleration,  $a = 2 \text{ ms}^{-2}$ ,

$t = 10$  s

Using equation,  $v = u + at$ , we get

$$v = 0 + 2 \times 10 = 20 \text{ m/s}$$

The final velocity,  $v = 20 \text{ m/s}$

At time,  $t = 11$  sec, the horizontal component of the velocity in the absence of the air resistance remains unchanged

$$V_x = 20 \text{ m/s}$$

The vertical component of the velocity is given by the equation

$$V_y = u + a_y t$$

Here  $t = 11 - 10 = 1$  s and  $a_y = a = 10 \text{ m/s}^2$

Therefore, the resultant velocity  $v$  of the stone is

$$v = (v_x^2 + v_y^2)^{1/2}$$

$$v = (20^2 + 10^2)^{1/2}$$

$$v = 22.36 \text{ m/s}$$

$$\tan \theta = v_y/v_x = 10/20 = 1/2 = 0.5$$

$$\theta = \tan^{-1}(0.5) = 26.56^\circ \text{ from the horizontal}$$

(b) When the stone is dropped from the truck, the horizontal force on the stone is zero. The only acceleration of the stone is that due to gravity which is equal to  $10 \text{ m/s}^2$  and it acts vertically downwards.

**12. A bob of mass  $0.1 \text{ kg}$  hung from the ceiling of a room by a string  $2 \text{ m}$  long is set into oscillation. The speed of the bob at its mean position is  $1 \text{ ms}^{-1}$ . What is the trajectory of the bob if the string is cut when the bob is (a) at one of its extreme positions, (b) at its mean position?**

**Solution:**

(a) When the bob is at one of its extreme positions, the velocity is zero. So, if the string is cut the bob will fall vertically downward under the force of its weight  $F = mg$ .

(b) At its mean position the bob has a horizontal velocity. If the string is cut, the bob will behave like a projectile and will fall on the ground after taking a parabolic path.

**13. A man of mass  $70 \text{ kg}$ , stands on a weighing machine in a lift, which is moving**

(a) upwards with a uniform speed of  $10 \text{ ms}^{-1}$ .

(b) downwards with a uniform acceleration of  $5 \text{ ms}^{-2}$ .

(c) upwards with a uniform acceleration of  $5 \text{ ms}^{-2}$ .

What would be the readings on the scale in each case?

(d) What would be the reading if the lift mechanism failed and it hurtled down freely under gravity?

**Solution:**

Mass of the man,  $m = 70 \text{ kg}$ ,

$$g = 10 \text{ m/s}^2$$

The weighing machine in each case measures the reaction  $R$ , i.e., the apparent weight.

(a) When the lift moves upwards with a uniform speed of  $10 \text{ m/s}$ , its acceleration =  $0$ .

$$R = mg = 70 \times 10 = 700 \text{ N}$$

(b) Lift moving downwards with  $a = 5 \text{ ms}^{-2}$

Using Newton's second law of motion, the equation of motion can be written as

$$R + mg = ma$$

$$R = m(g - a) = 70(10 - 5) = 350 \text{ N}$$

(c) Lift moving upwards with  $a = 5 \text{ ms}^{-2}$

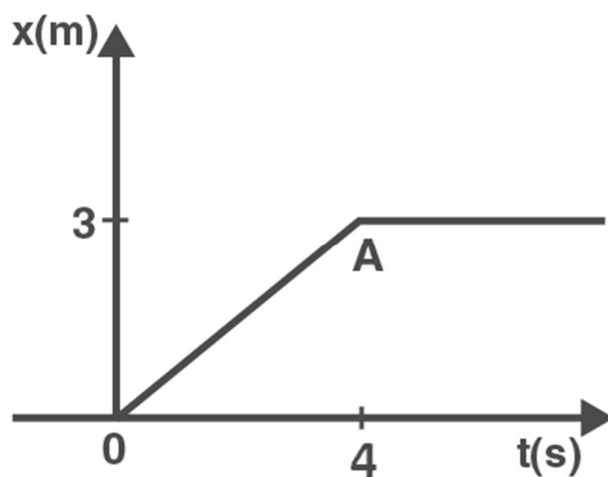
$$R = m(g + a) = 70(10 + 5) = 1050 \text{ N}$$

(d) If the lift were to come down freely under gravity, downward  $a = g$

$$\therefore R = m(g - a) = m(g - g) = 0$$

The man will be in a state of weightlessness

14. Figure shows the position-time graph of a particle of mass  $4 \text{ kg}$ . What is the (a) force on the particle for  $t < 0$ ,  $t > 4 \text{ s}$ ,  $0 < t < 4 \text{ s}$ ? (b) impulse at  $t = 0$  and  $t = 4 \text{ s}$ ? (Consider one-dimensional motion only).



**Solution:**

When  $t < 0$ , the distance covered by the particle is zero. Therefore, the force on the particle is zero.

When  $0 < t < 4 \text{ s}$ , the particle is moving with a constant velocity. Therefore, the force will be zero.

When  $t > 4 \text{ s}$ , the particle remains at a constant distance. Therefore, the force of the particle will be zero.



Impulse at  $t = 0$ .

Here,  $u = 0$

$$v = \frac{3}{4} = 0.75 \text{ m/s}$$

$$M = 4 \text{ kg}$$

Impulse = total change in momentum =  $mv - mu = m(v - u)$

$$= 4(0 - 0.75) = -3 \text{ kg m/s}$$

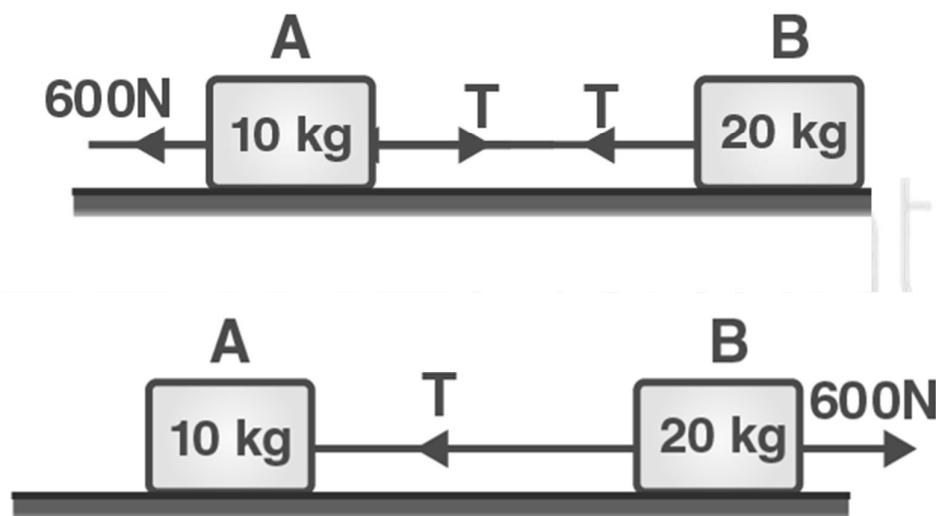
Impulse at  $t = 4\text{s}$

$$u = 0.75 \text{ m/s}, v = 0$$

$$\text{Impulse} = m(v - u) = 4(0 - 0.75) = -3 \text{ kg m/s}$$

15. Two bodies of masses 10 kg and 20 kg respectively kept on a smooth, horizontal surface are tied to the ends of a tight string. A horizontal force  $F = 600 \text{ N}$  is applied to (i) A, (ii) B along the direction of string. What is the tension in the string in each case?

Solution:



Given,

Mass of the body A,  $m_1 = 10 \text{ kg}$

Mass of the body B,  $m_2 = 20 \text{ kg}$

Horizontal force = 600 N

Total mass of the system,  $m = m_1 + m_2 = 30 \text{ kg}$

Applying Newton's second law of motion, we have

$$F = ma$$

$$a = F/m = 600/30 = 20 \text{ m/s}^2$$

(i) When the force is applied on A (10 kg)

$$F - T = m_1 a$$

$$T = F - m_1 a$$

$$T = 600 - (10 \times 20)$$

$$= 600 - 200 = 400 \text{ N}$$

(ii) When the force is applied on B (20 kg)

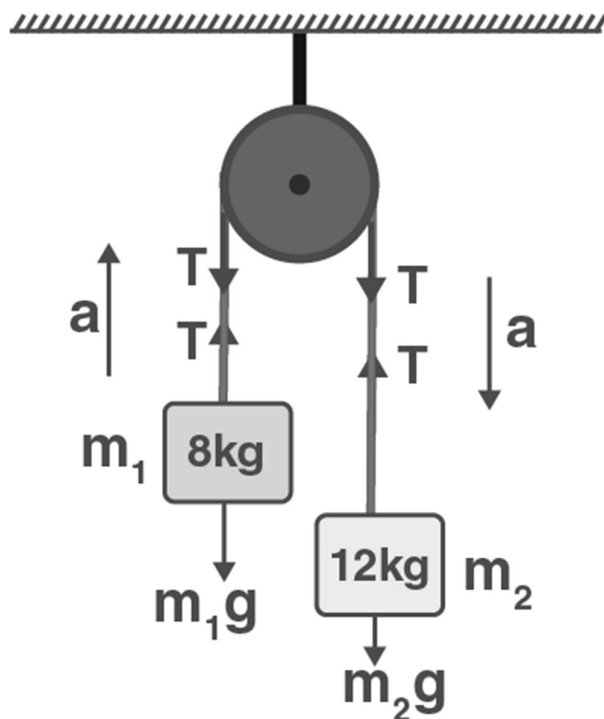
$$F - T = m_2 a$$

$$T = F - m_2 a$$

$$= 600 - (20 \times 20) = 200 \text{ N}$$

16. Two masses 8 kg and 12 kg are connected at the two ends of a light inextensible string that goes over a frictionless pulley. Find the acceleration of the masses and the tension in the string when the masses are released.

Solution:



Given,

Smaller mass,  $m_1 = 8 \text{ kg}$

Larger mass,  $m_2 = 12 \text{ kg}$

Tension in the string =  $T$

The heavier mass  $m_2$  will move downwards and the smaller mass  $m_1$  will move upwards.

Applying Newton's second law,

For mass  $m_1$ :

$$T - m_1g = m_1a \text{ --- (1)}$$

For mass  $m_2$ :

$$m_2g - T = m_2a \text{ --- (2)}$$

Add (1) and (2)

$$(m_2 - m_1)g = (m_1 + m_2)a$$

$$a = (m_2 - m_1)g / (m_1 + m_2)$$

$$= [(12 - 8)/(12 + 8)] \times 10 = (4/20) \times 10 = 2 \text{ m/s}$$

Therefore, acceleration of the mass is  $2 \text{ m/s}^2$

Substituting this value of acceleration in equation (ii) we get

$$m_2g - T = m_2a$$

$$m_2g - T = m_2 [(m_2 - m_1)g / (m_1 + m_2)]$$

$$= 2m_1m_2g / (m_1 + m_2)$$

$$T = 2 \times 12 \times 8 \times 10 / (12 + 8)$$

$$T = 96 \text{ N}$$

Therefore, the tension on the string is 96 N

**17. A nucleus is at rest in the laboratory frame of reference. Show that if it disintegrates into two smaller nuclei, the products must move in opposite directions.**

**Solution:**

Let  $m_1, m_2$  be the masses of the two daughter nuclei and  $v_1, v_2$  be their respective velocities of the daughter nuclei. Let  $m$  be the mass of the parent nucleus.

Total linear momentum after disintegration =  $m_1v_1 + m_2v_2$ .

Before disintegration, the nucleus is at rest. Therefore, its linear momentum before disintegration is zero.

Applying the law of conservation of momentum,

Total linear momentum before disintegration = Total linear momentum after disintegration

$$0 = m_1v_1 + m_2v_2$$

$$v_1 = -m_2v_2 / m_1$$

The negative sign indicates  $v_1$  and  $v_2$  are in opposite directions.

**18. Two billiard balls, each of mass  $0.05 \text{ kg}$ , moving in opposite directions with speed  $6 \text{ ms}^{-1}$  collide and rebound with the same speed. What is the impulse imparted to each ball due to the other?**

**Solution:**

Mass of each ball =  $0.05 \text{ kg}$

Initial velocity of each ball =  $6 \text{ m/s}$

The initial momentum of each ball before the collision

$$= 0.05 \times 6 = 0.3 \text{ kg m/s}$$

After the collision, the balls change in the direction of motion without a change in the magnitude of the velocity

Final momentum after collision of the first ball =  $-0.05 \times 6 = -0.3 \text{ kg m/s}$

Final momentum after collision of the second ball =  $0.3 \text{ kg m/s}$

Impulse imparted to the first ball =  $(-0.3) - (0.3) = -0.6 \text{ kg m/s}$

Impulse imparted to the second ball =  $(0.3) - (-0.3) = 0.6 \text{ kg m/s}$

The two impulses are opposite in direction.

**19. A shell of mass  $0.020 \text{ kg}$  is fired by a gun of mass  $100 \text{ kg}$ . If the muzzle speed of the shell is  $80 \text{ ms}^{-1}$ . What is the recoil speed of the gun?**

**Solution:**

Mass of the shell,  $m = 0.020 \text{ kg}$

Mass of the gun,  $M = 100 \text{ Kg}$

Speed of the shell =  $80 \text{ m/s}$

The initial velocity of the shell and the gun is zero. So, the initial momentum of the system is zero.

Applying the law of conservation of momentum, the initial momentum is equal to the final momentum.

So,

$$0 = mv - MV$$

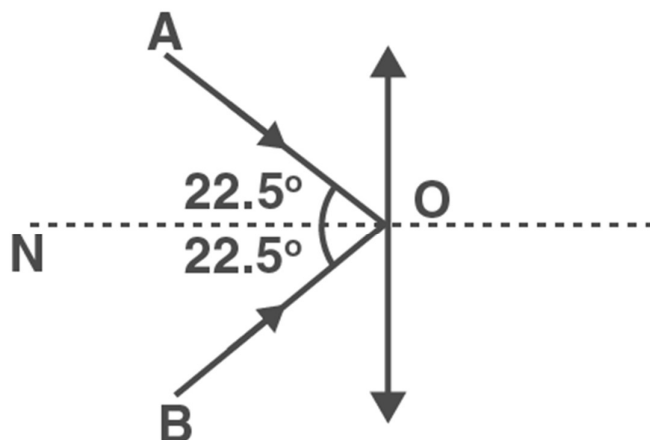
Recoil speed of the gun,  $v = mv/M$

$$v = (0.020 \times 80)/100$$

$$v = 0.016 \text{ m/s}$$

**20. A batsman deflects a ball by an angle of  $45^\circ$  without changing its initial speed which is equal to  $54 \text{ km/h}$ . What is the impulse imparted to the ball? (Mass of the ball is  $0.15 \text{ kg}$ .)**

**Solution:**



Velocity of the ball = 54 km/h

The ball is deflected back such that the total angle =  $45^\circ$

The initial momentum of the ball is  $m \cos \theta = (0.15 \times 54 \times 1000 \times \cos 22.5^\circ) / 3600$

$= 0.15 \times 15 \times 0.9239$  along NO

Final momentum of the ball =  $m \cos \theta$  along ON

Impulse = change in the momentum =  $m \cos \theta - (-m \cos \theta) = 2m \cos \theta = 2 \times 0.15 \times 15 \times 0.9239 = 4.16 \text{ kg.m/s}$

21. A stone of mass 0.25 kg tied to the end of a string is whirled round in a circle of radius 1.5 m with a speed of 40 rev./min in a horizontal plane. What is the tension in the string? What is the maximum speed with which the stone can be whirled around if the string can withstand a maximum tension of 200 N?

**Solution:**

Mass of the stone = 0.25 kg

Radius,  $r = 1.5 \text{ m}$

Number of revolution in a second,  $n = 40/60 = (2/3) \text{ rev/sec}$

The angular velocity,  $\omega = 2\pi n = 2 \times 3.14 \times (2/3)$

The tension on the string provides the necessary centripetal force

$$T = m\omega^2 r$$

$$T = 0.25 \times 1.5 \times [2 \times 3.14 \times (2/3)]^2$$

$$= 6.57 \text{ N}$$

Maximum tension on the string,  $T_{\max} = 200 \text{ N}$

$$T_{\max} = mv_{\max}^2 / r$$

$$v_{\max}^2 = (T_{\max} \times r) / m$$

$$= (200 \times 1.5)/0.25 = 1200$$

$$v_{\max} = 34.6 \text{ m/s}$$

Therefore, the maximum speed of the stone is 34.64 m/s

22. If in problem 21, the speed of the stone is increased beyond the maximum permissible value, and the string breaks suddenly, which of the following correctly describes the trajectory of the stone after the string breaks?

- (a) the stone moves radially outwards
- (b) the stone flies off tangentially from the instant the string breaks
- (c) the stone flies off at an angle with the tangent whose magnitude depends on the speed of the particle

**Solution: (b)**

At each point of the circular motion, the velocity will be tangential. If the string breaks suddenly the stone moves in the tangential direction according to Newton's first law of motion.

23. Explain why

- (a) a horse cannot pull a cart and run in empty space
- (b) passengers are thrown forward from their seats when a speeding bus stops suddenly
- (c) it is easier to pull a lawnmower than to push it
- (d) a cricketer moves his hands backwards while holding a catch

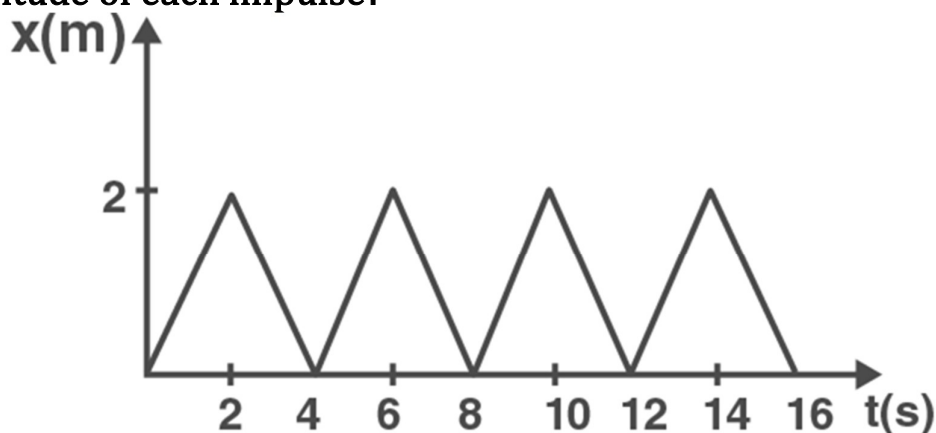
**Solution:**

1. The horse pushes the ground with a certain force when it pulls the cart. By applying the third law of motion, the ground will exert an equal and opposite reaction force upon the feet of the horse. This causes the horse to move forward. In empty space, the horse will not experience a reaction force. Therefore, the horse cannot pull the cart in empty space.
2. Due to inertia of motion. When a bus stops all of a sudden, the lower part of a person's body that is in contact with the seat comes to rest suddenly but the upper part will continue to be in motion. As a result, the person's upper half of the body is thrown forward in the direction of the motion of the bus.
3. When the lawnmower is pulled, the vertical component of the applied force acts upwards. This reduces the effective weight of the lawnmower. When the lawn mower is pushed, the vertical component acts in the direction of the weight of the mower.

Therefore, there is an increase in the weight of the mower. So, it is easier to pull a lawnmower than to push it.

4. When the batsman hits the ball, the ball will have a large momentum. When he moves his hands backwards the time of impact is increased contact, so the force is reduced.

24. Figure shows the position-time graph of a particle of mass 0.04 kg. Suggest a suitable physical context for this motion. What is the time between two consecutive impulses received by the particle? What is the magnitude of each impulse?



**Solution:**

This graph could be of a ball rebounding between two walls separated by a distance of 2 cm. The ball rebounds between the walls every 2 seconds with a uniform velocity.

$$\text{Velocity} = \text{displacement} / \text{time} = (2 \times 10^{-2}) / 2 = 0.01 \text{ m/s}$$

$$\text{Initial momentum} = mu = 0.04 \times 0.01 = 4 \times 10^{-4} \text{ kgm/s}$$

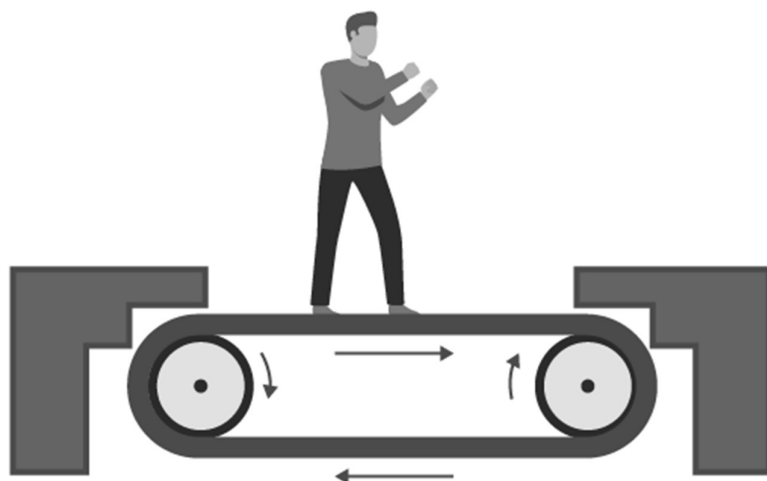
$$\text{Final momentum} = mv = 0.04 \times (-0.01) = -4 \times 10^{-4} \text{ kgm/s}$$

$$\text{Magnitude of impulse} = \text{Change in momentum}$$

$$= (4 \times 10^{-4}) - (-4 \times 10^{-4}) = 8 \times 10^{-4} \text{ kgm/s}$$

The time between two consecutive impulses is 2 sec, so the ball receives an impulse every 2 seconds.

25. Figure shows a man standing stationary with respect to a horizontal conveyor belt that is accelerating with  $1 \text{ ms}^{-2}$ . What is the net force on the man? If the coefficient of static friction, between the man's shoes and the belt is 0.2, up to what acceleration of the belt can the man continue to be stationary relative to the belt? (Mass of the man = 65 kg.)

**Solution:**

Here acceleration of conveyor belt  $a = 1 \text{ ms}^{-2}$ ,

Coefficient of static friction,  $\mu_s = 0.2$

mass of man,  $m = 65 \text{ kg}$

Net Force  $= ma = 65 \times 1 = 65 \text{ N}$

This net force is due to the friction between the belt and the man

Maximum static friction  $= \mu_s N$

Beyond that, the man starts moving relative to the belt as per the pseudo force

At maximum static friction

$$\mu_s N = \mu_s .mg = ma_{\text{max}}$$

$$a_{\text{max}} = \mu_s g = 0.2 \times 10 = 2 \text{ m/s}^2$$

**26. A stone of mass  $m$  tied to the end of a string is revolving in a vertical circle of radius  $R$ . The net force at the lowest and highest points of the circle directed vertically downwards are: (choose the correct alternative).**

Lowest Point

Highest Point

(a)  $mg - T_1$

$mg + T_2$

(b)  $mg + T_1$

$mg - T_2$

(c)  $mg + T_1 - (mv_1^2) / R$

$mg - T_2 + (mv_1^2) / R$

(d)  $mg - T_1 - (mv_1^2) / R$

$mg + T_2 + (mv_1^2) / R$

$T_1$  and  $v_1$  denote the tension and speed at the lowest point.  $T_2$  and  $v_2$  denote corresponding values at the highest point.

**Solution: (a)**

The net force at the lowest point is  $(mg - T_1)$  and the net force at the highest point is  $(mg + T_2)$ . Therefore option (a) is correct.



Since  $mg$  and  $T_1$  are in mutually opposite directions at the lowest point and  $mg$  and  $T_2$  are in the same direction at the highest point.

**27. A helicopter of mass 1000 kg rises with a vertical acceleration of  $15 \text{ ms}^{-2}$ . The crew and the passengers weigh 300 kg. Give the magnitude and direction of**

**(a) force on the floor by the crew and passengers**

**(b) the action of the rotor of the helicopter on surrounding air**

**(c) force on the helicopter due to the surrounding air**

**Solution:**

Mass of helicopter = 1000 kg

Crew and passengers weight = 300 kg

Vertical acceleration,  $a = 15 \text{ ms}^{-2}$  and  $g = 10 \text{ ms}^{-2}$

The total mass of the system,  $m_i = 1000 + 300 = 1300 \text{ Kg}$

**(a) Force on the floor of the helicopter by the crew and passengers**

$$R - mg = ma$$

$$= m(g+a)$$

$$= m(g+a) = 300(10+15) \text{ N} = 7500 \text{ N}$$

**(b) Action of the rotor of the helicopter on surrounding air is due to the mass of the helicopter and the passengers.**

$$R' - m_i g = m_i a$$

$$R' = m_i(g+a)$$

$$= 1300 \times (10+15) = 32500 \text{ N}$$

This force acts vertically downwards

**(c) Force on the helicopter due to the surrounding air is the reaction of the force applied by the rotor on the air. As action and reaction are equal and opposite, therefore, the force of reaction,  $F = 32500 \text{ N}$ . This force acts vertically upwards.**

**28. A stream of water flowing horizontally with a speed of  $15 \text{ m/s}$  pushes out of a tube of cross-sectional area  $10^{-2} \text{ m}^2$  and hits at a vertical wall nearby. What is the force exerted on the wall by the impact of water, assuming that it does not rebound?**

**Solution:**

Speed of water flowing,  $v = 15 \text{ m/s}$

Cross-sectional area of the tube,  $A = 10^{-2} \text{ m}^2$

In one second, the distance travelled is equal to the velocity  $v$

Density of water,  $\rho = 1000 \text{ kgm}^{-3}$

Mass of water hitting the wall per second =  $\rho \times A \times v$

$$= 1000 \text{ kgm}^{-3} \times 10^{-2} \text{ m}^2 \times 15 \text{ m/s} = 150 \text{ kg/s}$$

Force exerted on the wall because of the impact of water = momentum loss of water per second

$$= \text{Mass} \times \text{velocity}$$

$$= 150 \text{ kg/s} \times 15 \text{ m/s} = 2250 \text{ N}$$

**29. Ten one rupee coins are put on top of one another on a table. Each coin has a mass of  $m$  kg. Give the magnitude and direction of**

**(a) the force on the 7th coin (counted from the bottom) due to all coins above it**

**(b) the force on the 7th coin by the eighth coin**

**(c) the reaction of the sixth coin on the seventh coin**

**Solution:**

(a) The force on the 7th coin is due to the weight of the three coins kept above it.

Weight of one coin is  $mg$ . So the weight of three coins is  $3mg$

Force exerted on the 7th coin is  $(3mg)\text{N}$  and the force acts vertically downwards.

(b) The eighth coin is already under the weight of two coins above it and it has its own weight too. Hence, the force on the 7th coin due to the 8th coin will be the same as the force on the 7th coin by the three coins above it.

Therefore, the force on the 7th coin by the 8th coin is  $(3mg)\text{N}$  and the force act vertically downwards.

(c) The sixth coin is under the weight of four coins above it and experiences a downward force due to the four coins.

The total downward force on the 6th coin is  $4mg$

Applying Newton's third law of motion, the 6th coin will exert a reaction force upwards. Therefore, the force exerted by the 6th coin on the 7th coin is equal to  $4mg$  and acts in the upward direction.

**30. An aircraft executes a horizontal loop at a speed of  $720 \text{ km/h}$  with its wings banked at  $15^\circ$ . What is the radius of the loop?**

**Solution:**

The speed of the aircraft executing the horizontal loop =  $720 \text{ km/h} = 720 \times (5/18) = 200 \text{ m/s}$

The angle of banking =  $15^\circ$

$\tan\theta = v^2/rg$ , we have

$$r = v^2/g \tan\theta = (200 \times 200)/(10 \times \tan 15^\circ) = (200 \times 200)/(10 \times 0.2679) = 14931 \text{ m}$$

31. A train runs along an unbanked circular track of radius of 30 m at a speed of 54 km/h. The mass of the train is  $10^6$  kg. What provides the centripetal force required for this purpose the engine or the rails? What is the angle of banking required to prevent wearing out of the rail?

**Solution:**

Radius of the track = 30 m

Velocity of the train = 54 km/h =  $54 \times (5/18) = 15$  m/s

Mass of the train =  $10^6$  kg

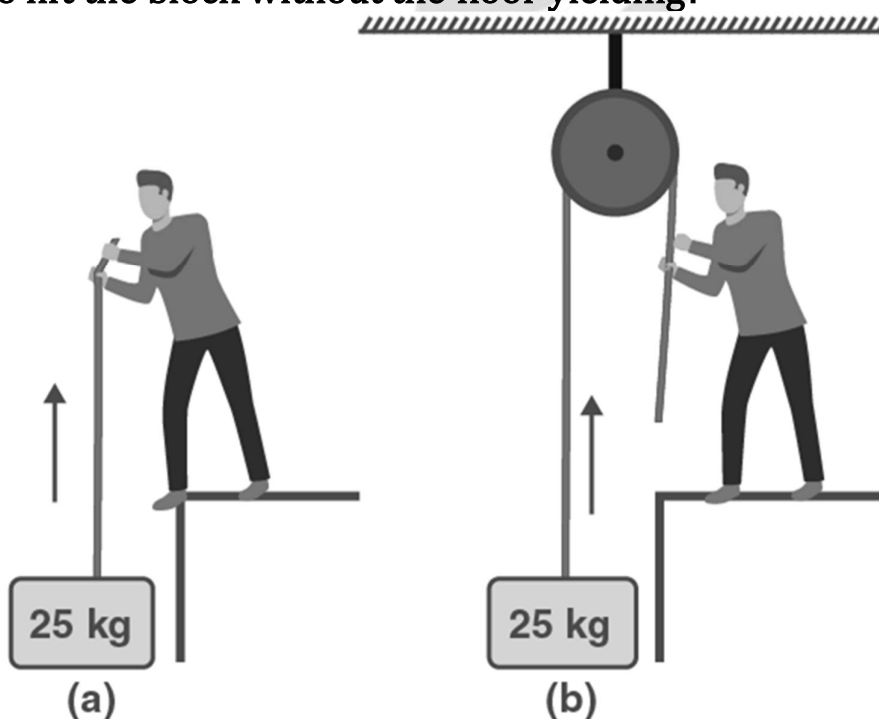
The required centripetal force is provided by the force of lateral friction due to the rails on the wheels of the train

The angle of banking required to prevent wearing out of the rails

$$\tan\theta = v^2/rg = (15 \times 15) / (30 \times 10) = 0.75$$

$$\theta = \tan^{-1}(0.75) = 37^\circ$$

32. A block of mass 25 kg is raised by a 50 kg man in two different ways as shown in Fig. What is the action on the floor by the man in the two cases? If the floor yields to a normal force of 700 N, which mode should the man adopt to lift the block without the floor yielding?



**Solution:**

Mass of the block = 25 kg

Mass of the man = 50 kg

Acceleration due to gravity =  $10 \text{ m/s}^2$

Weight of the block,  $F = 25 \times 10 = 250 \text{ N}$

Weight of the man,  $W = 50 \times 10 = 500 \text{ N}$

In the 1st case, the man lifts the block directly by applying an upward force of  $250 \text{ N}$  (same as the weight of the block)

According to Newton's third law of motion, there will be a downward reaction on the floor. The action on the floor by the man.

$$= 500 \text{ N} + 250 \text{ N} = 750 \text{ N}.$$

In the 2nd case, the man applies a downward force of  $25 \text{ kg wt.}$  According to Newton's third law of motion, the reaction is in the upward direction.

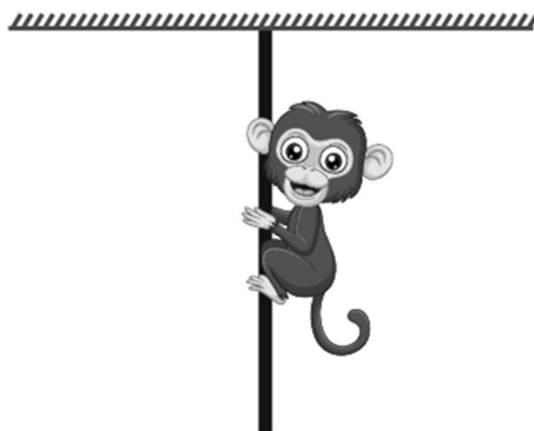
In this case, action on the floor by the man

$$= 500 \text{ N} - 250 \text{ N} = 250 \text{ N}.$$

Therefore, the man should adopt the second method.

**33. A monkey of mass  $40 \text{ kg}$  climbs on a rope (Fig.) which can stand a maximum tension of  $600 \text{ N}$ . In which of the following cases will the rope break, when the monkey**

- (a) climbs up with an acceleration of  $6 \text{ ms}^{-2}$**
  - (b) climbs down with an acceleration of  $4 \text{ ms}^{-2}$**
  - (c) climbs up with a uniform speed of  $5 \text{ ms}^{-1}$**
  - (d) falls down the rope nearly freely under gravity**
- (Ignore the mass of the rope).**



**Solution:**

Mass of the monkey =  $40 \text{ kg}$

Maximum tension the rope can withstand,  $T_{\text{max}} = 600 \text{ N}$

(a) When the monkey climbs up with an acceleration of  $6 \text{ m/s}^2$ ,

Tension  $T - mg = ma$

$$T = m(g+a)$$

$$T = 40(10 + 6)$$

$$= 640 \text{ N}$$

Since  $T > T_{\max}$ , the rope will break

(b) When the monkey climbs down with the acceleration of  $4\text{m/s}^2$

$$mg - T = ma$$

$$T = mg - ma = m(g - a)$$

$$= 40(10 - 4) = 240 \text{ N}$$

Since  $T < T_{\max}$ , the rope will not break

(c) When the monkey climbs with a uniform speed  $5\text{m/s}$ . The acceleration will be zero. The equation of motion is

$$T - mg = ma$$

$$T - mg = 0$$

$$T = mg = 40 \times 10 = 400 \text{ N}$$

Since  $T < T_{\max}$ , the rope will not break

(d) When the monkey falls freely, the acceleration of the monkey will be equal to the acceleration due to gravity

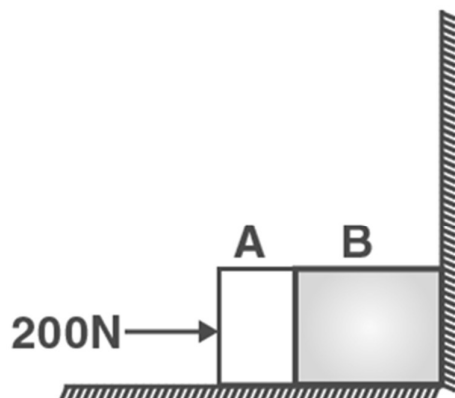
The equation of motion is written as

$$mg + T = mg$$

$$T = m(g-g) = 0$$

Since  $T < T_{\max}$ , the rope will not break

**34. Two bodies A and B of masses 5 kg and 10 kg in contact with each other rest on a table against a rigid wall (Fig.). The coefficient of friction between the bodies and the table is 0.15. A force of 200 N is applied horizontally to A. What are (a) the reaction of the partition (b) the action-reaction forces between A and B? What happens when the wall is removed? Does the Solution to (b) change, when the bodies are in motion? Ignore the difference between  $\mu_s$  and  $\mu_k$ .**

**Solution:**

Mass of the body,  $m_A = 5 \text{ Kg}$

Mass of the body,  $m_B = 10 \text{ kg}$

Applied Force = 200 N

Coefficient of friction between the bodies and the table  $\mu_s = 0.15$

(a) The force of friction is given by the relation:

$$f_s = \mu(m_A + m_B)g$$

$$= 0.15 (5 + 10) \times 10$$

$$= 1.5 \times 15 = 22.5 \text{ N, towards left}$$

Therefore, the net force on the partition is  $200 - 22.5 = 177.5 \text{ N}$  rightward

According to Newton's third law, action and reaction are in the opposite direction

Therefore, the reaction of the partition will be  $177.5 \text{ N}$ , in the leftward direction

(b) Force of friction on mass A

$$F_A = \mu m_A g$$

$$= 0.15 \times 5 \times 10 = 7.5 \text{ N leftward}$$

The net force exerted by mass A on mass B =  $200 - 7.5 = 192.5 \text{ N}$  rightwards

An equal amount of reaction force will be applied on mass A by B, i.e.,  $192.5 \text{ N}$  acting leftward

When the wall is removed, the two bodies move in the direction of the applied force

The net force acting on the moving system =  $177.5 \text{ N}$

The equation of motion for the system of acceleration  $a$ , can be written as

$$\text{Net force} = (m_A + m_B) a$$

$$a = \text{Net force} / (m_A + m_B)$$

$$= 177.5 / (5 + 10) = 177.5 / 15 = 11.83 \text{ m/s}^2$$

Net force causing mass A to move

$$F_A = m_A a = 5 \times 11.83 = 59.15 \text{ N}$$

$$\text{Net force exerted by the mass A on mass B} = 192.5 - 59.15 = 133.35 \text{ N}$$

This force acts in the direction of motion. As per Newton's third law of motion, an equal amount of force will be exerted by mass B on mass A, i.e., 133.3N, acting opposite to the direction of motion.

**35. A block of mass 15 kg is placed on a long trolley. The coefficient of static friction between the block and the trolley is 0.18. The trolley accelerates from rest with  $0.5 \text{ ms}^{-2}$  for 20 s and then moves with uniform velocity.**

**Discuss the motion of the block as viewed by (a) a stationary observer on the ground (b) an observer moving with the trolley.**

**Solution:**

Mass of the block = 15 kg

Coefficient of static friction between the block and the trolley,  $\mu = 0.18$

Acceleration of the trolley =  $0.5 \text{ m/s}^2$

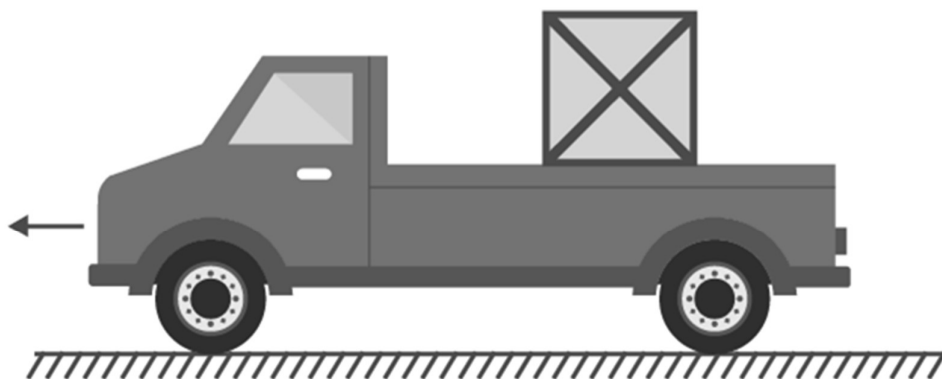
(a) Force experienced by block,  $F = ma = 15 \times 0.5 = 7.5 \text{ N}$ , this force acts in the direction of motion of the trolley

Force of friction,  $F_f = \mu mg = 0.18 \times 15 \times 10 = 27 \text{ N}$

Force experienced by the block will be less than the friction. So the block will not move. So, for a stationary observer on the ground, the block will be stationary.

(b) The observer moving with the trolley has an accelerated motion. He forms a non-inertial frame in which Newton's laws of motion are not applicable. The trolley will be at rest for the observer moving with the trolley

**36. The rear side of a truck is open and a box of 40 kg mass is placed 5 m away from the open end as shown in Fig. The coefficient of friction between the box and the surface below it is 0.15. On a straight road, the truck starts from rest and accelerates with  $2 \text{ ms}^{-2}$ . At what distance from the starting point does the box fall off the truck? (Ignore the size of the box).**

**Solution:**

Force experienced by box,  $F = ma = 40 \times 2 = 80 \text{ N}$

Frictional force  $F_f = \mu mg = 0.15 \times 40 \times 10 = 60 \text{ N}$

Net force  $= F - F_f = 80 - 60 = 20 \text{ N}$

Backward acceleration produced in the box,  $a = 20/40$  (Net Force/m)

$a = 0.5 \text{ ms}^{-2}$

If  $t$  is time taken by the box to travel  $s = 5$  metre and fall off the truck then from

$$S = ut + \frac{1}{2} at^2$$

$$5 = 0 \times t + \left(\frac{1}{2}\right) \times 0.5 t^2$$

$$t = \sqrt{(5 \times 2 / 0.5)} = 4.47 \text{ s}$$

If the truck travels a distance  $x$  during this time, then again from

$$S = ut + \frac{1}{2} at^2$$

$$x = 0 \times 4.47 + \left(\frac{1}{2}\right) \times 2(4.47)^2 = 19.98 \text{ m}$$

**37. A disc revolves with a speed of  $33\frac{1}{3}$  rpm and has a radius of 15 cm. Two coins are placed at 4 cm and 14 cm away from the centre of the record. If the coefficient of friction between the coins and record is 0.15, which of the coins will revolve with the record?**

**Solution:**

Speed of revolution of the disc  $= 33\frac{1}{3} \text{ rpm} = 100/3 \text{ rpm} = 100 / (3 \times 60)$

$\text{rps} = 5/9 \text{ rps}$

$$\omega = 2\pi v = 2 \times (22/7) \times (5/9) = 220/63 \text{ rad/s}$$

The coins revolve with the disc, the centripetal force is provided by the frictional force  $mv^2/r \leq \mu mg$  ———(1)

As  $v = r\omega$ , equation (1) becomes  $mr\omega^2/r \leq \mu mg$

$$r \leq \mu g / \omega^2 = (0.15 \times 10) / (220/63)^2 = 12 \text{ cm}$$



For coin A,  $r = 4 \text{ cm}$

The condition ( $r \leq 12$ ) is satisfied for coin placed at 4cm, so coin A will revolve with the disc

For coin B,  $r = 14 \text{ cm}$

The condition ( $r \leq 12$ ) is not satisfied for the coin placed at 14cm, so coin B will not revolve with the disc.

**38. You may have seen in a circus a motorcyclist driving in vertical loops inside a 'death well' (a hollow spherical chamber with holes, so the spectators can watch from outside). Explain clearly why the motorcyclist does not drop down when he is at the uppermost point, with no support from below. What is the minimum speed required at the uppermost position to perform a vertical loop if the radius of the chamber is 25 m?**

**Solution:**

When the motorcyclist is at the uppermost point of the death-well, the normal reaction  $R$  on the motorcyclist by the ceiling of the chamber acts downwards. His weight  $mg$  also acts downwards. The outward centrifugal force acting on the motorcyclist is balanced by these two forces.

$$R + mg = mv^2/r \text{ ———(1)}$$

Here,  $v$  is the velocity of the motorcyclist

$m$  is the mass of the motorcyclist and the motorcycle

Because of the balance between the forces, the motorcyclist does not fall

The minimum speed required at the uppermost position to perform a vertical loop is given by the equation (1) when  $R = 0$

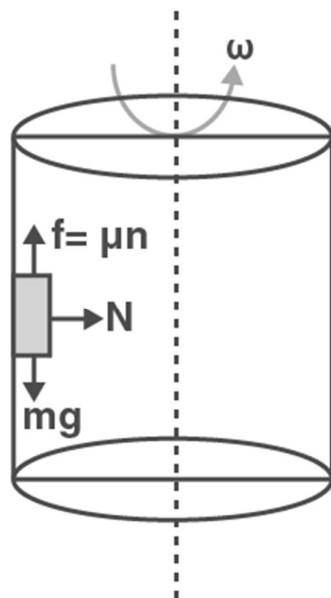
$$mg = mv_{\min}^2/r$$

$$V_{\min}^2 = gr$$

$$V_{\min} = \sqrt{10 \times 25} = 15.8 \text{ m/s}$$

**39. A 70 kg man stands in contact against the inner wall of a hollow cylindrical drum of radius 3 m rotating about its vertical axis with 200 rev/min. The coefficient of friction between the wall and his clothing is 0.15. What is the minimum rotational speed of the cylinder to enable the man to remain stuck to the wall (without falling) when the floor is suddenly removed?**

**Solution:**



Mass of the man,  $m = 70 \text{ kg}$

Radius of the drum,  $r = 3 \text{ m}$

Coefficient of friction between the wall and his clothing,  $\mu = 0.15$

Number of revolution of hollow cylindrical drum  $= 200 \text{ rev/min} = 200/60 = 10/3 \text{ rev/s}$

The centripetal force required is provided by the normal  $N$  of the wall on the man

$$N = mv^2/R = m\omega^2 R$$

When the floor revolves, the man sticks to the wall of the drum. Hence, the weight of the man ( $mg$ ) acting downward is balanced by the frictional force acting vertically upwards.

The man will not fall if  $mg \leq$  limiting frictional force  $f_e (\mu N)$

$$mg \leq \mu N$$

$$mg \leq \mu (m\omega^2 R)$$

$$\omega^2 \geq g/R\mu$$

Therefore, for minimum rotational speed of the cylinder

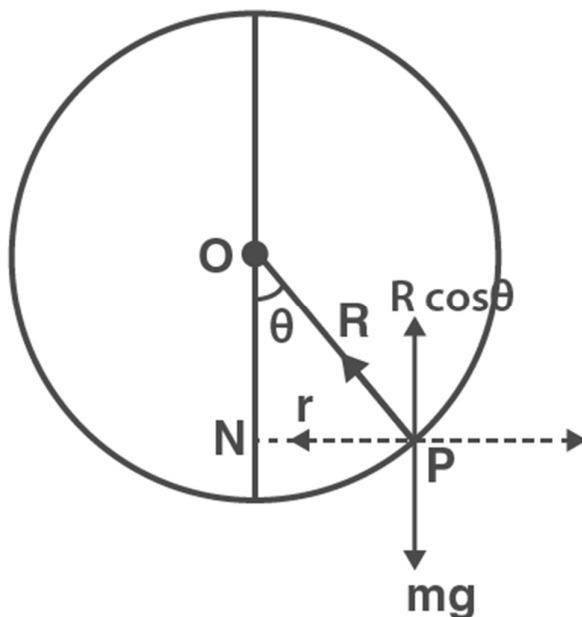
$$\omega^2 = g/R\mu = 10/(0.15 \times 3) = 22.2$$

$$\omega = \sqrt{22.2} = 4.7 \text{ rad/s}$$

**40.** A thin circular loop of radius  $R$  rotates about its vertical diameter with an angular frequency  $\omega$ . Show that a small bead on the wire loop remains at its lowermost point for  $\omega \leq \sqrt{g/R}$ . What is the angle made by the radius

vector joining the centre to the bead with the vertically downward direction for  $\omega = \sqrt{2g/R}$ ? Neglect friction.

Solution:



Let  $\theta$  be the angle made by the radius vector joining the bead and the centre of the wire with the downward direction. Let,  $N$  be the normal reaction

$$mg = N \cos \theta \text{ --- (1)}$$

$$m r \omega^2 = N \sin \theta \text{ --- (2)}$$

$$\text{(or) } m (R \sin \theta) \omega^2 = N \sin \theta$$

$$m R \omega^2 = N$$

Substituting the value of  $N$  in (1)

$$mg = m R \omega^2 \cos \theta$$

$$\text{(or) } \cos \theta = g / R \omega^2 \text{ --- (3)}$$

As  $|\cos \theta| \leq 1$  the bead will remain at the lowermost point

$$g / R \omega^2 \leq 1 \text{ or } \omega \leq \sqrt{g/R}$$

For  $\omega = \sqrt{2g/R}$ , equation (3) becomes

$$\cos \theta = g / R \omega^2$$

$$\cos \theta = (g/R) (R/2g) = 1/2$$

$$\theta = 60^\circ$$

\*\*\*\*\*